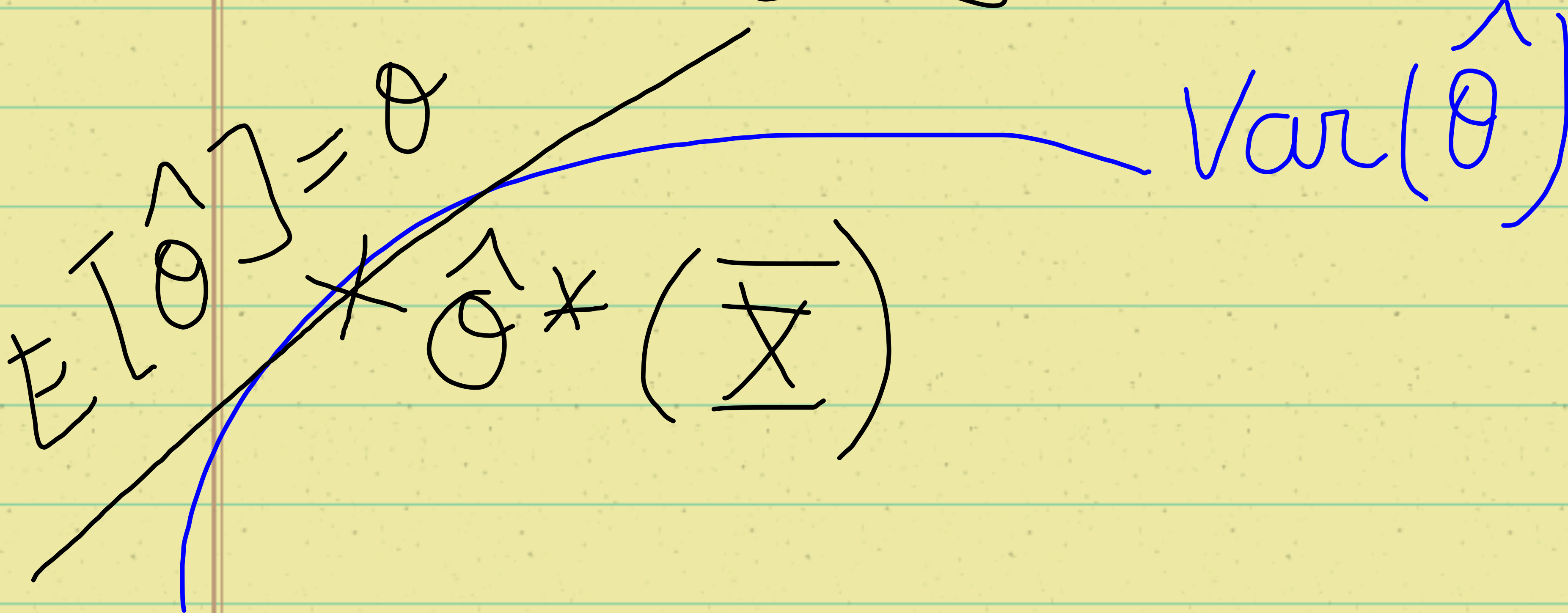
$$\min_{\hat{\theta}} E\left[(\hat{\theta}-\theta)^2\right]$$

$$\min \operatorname{Var}(\hat{\theta})$$

s.a. $E[\hat{\theta}] = \theta$



$$E[(\hat{\theta}-\theta)^2] = Var(\hat{\theta}) + (E[\hat{\theta}]-\theta)^2$$

$$E[\hat{\theta}^2] - \cancel{\theta^2} + E[\hat{\theta}]^2 - 2\theta E[\hat{\theta}]$$

$$+ \cancel{\theta^2}$$

$$Var(\hat{\theta}) \qquad (E[\hat{\theta}]-\theta)^2$$

$$\boxed{\begin{array}{c}\text{ERROR CUADRÁTICO} \\ \parallel \\ Var(\hat{\theta}) + \text{Cuadrado del sesgo}\end{array}}$$

$$\text{SESGO} = E[\hat{\theta}] - \theta$$

$\sigma = 3$€ $\quad E[S_x^2] = 9 = \sigma^2$

Tomamos una muestra

$s_x^2 = 6.3$ ¿Cuál es el sesgo?

$$E\left[(\hat{\theta}-\theta)^2\right] =$$

$$E\left[\hat{\theta}^2 - 2\hat{\theta}\theta + \theta^2\right] =$$

$$E\left[\hat{\theta}^2\right] - 2\theta E\left[\hat{\theta}\right] + \theta^2$$

Si $E\left[\hat{\theta}\right] = \theta$

$$E\left[\hat{\theta}^2\right] - 2\theta^2 + \theta^2 =$$

$$E\left[\hat{\theta}^2\right] - \theta^2 = Var(\hat{\theta})$$

Considerar estimadores insesgados $E\left[\hat{\theta}\right] = \theta$ de mínima varianza

Ex: $\hat{\theta} = \overline{X}$, $\hat{\theta} = S_x^2$

NO CONOCEMOS SI SON de mínima va

$$E\left[(\bar{X}-\mu)^2\right] = Var(\bar{X}) = \frac{\sigma^2}{n}$$

Si

$$Var(\bar{X}) = 0$$

$$\frac{1}{n}\sum(\bar{X}_i-\mu)^2 = 0$$

$$\bar{X}_i = \mu \quad \forall i$$

Buscar estimadores $\hat{\theta}(X_1, \ldots, X_n)$ de θ que cumplan

$$\boxed{E\left[(\hat{\theta}-\theta)^2\right] = Var(\hat{\theta})}$$

¿Qué estimadores cumplen esta condición?

ERROR ESTADÍSTICO

θ lo que quiero estimar

$\theta = \mu, \sigma$

$\hat{\theta}$ el estimador

$\hat{\theta} = \hat{\theta}(X_1, \ldots, X_n)$

$\hat{\theta} = \hat{\mu}(X_1, \ldots, X_n) = \overline{X}$

$\hat{\theta} = \hat{\sigma}(X_1, \ldots, X_n) = S_X$

ERROR

$$E\left[\left(\theta - \hat{\theta}(X_1, \ldots, X_n)\right)^2\right]$$

ERROR CUADRÁTICO MEDIO

$$E\left[(\mu - \overline{X})^2\right] = Var(\overline{X})$$

μ, σ

$$\hat{\mu}(x_1, \dots, x_{12}) = \frac{1}{12} \sum x_i$$

$$\hat{\sigma}(x_1, \dots, x_{12}) = \sqrt{\frac{1}{n-1} \sum (x_i - \bar{x})^2}$$

¿Cuanto me equivoco?

$$\text{error} = |\mu - \hat{\mu}(x_1, \dots, x_{12})|$$

$$|\mu - \hat{\mu}(X_1, \dots, X_{12})|$$

NO SE PUEDE CALCULAR

$$\left|\mu - \frac{1}{12} \sum_{1}^{12} X_i\right| =$$

$$|\mu - \bar{X}|$$

$$\underset{1/50}{|\mu - 32|}, \quad \underset{1/50}{|\mu - 27|}, \quad \dots \quad \underset{1/50}{|\mu - 42|}$$

$\overline{X}$

$$P(10 \le \overline{X} \le 12) =$$

$$P\left(\frac{10-10}{\frac{2}{\sqrt{12}}} \le N(0,1) \le \frac{12-10}{\frac{2}{\sqrt{12}}}\right)$$

Problema

INFORMACIÓN

Observo $x_1, \dots, x_{12}$

$X_1 \quad X_{12}$

¿Qué puedo decir de μ y σ?

$\hat{\mu}, \hat{\sigma}$ estimaciones de μ y σ

TEMA 2: ESTIMACIÓN PUNTUAL

$\underline{X}$ ≡ rendimiento mensual de una acción

$\underline{X}_1, \underline{X}_2, \ldots, \underline{X}_{12}$

$\boxed{x_1 \;,\; x_2, \qquad , x_{12}}$

$\bar{x} = \frac{1}{12} \sum x_i \qquad \bar{\underline{X}} = \frac{1}{12} \sum X_i$

$\underline{X} \overset{d}{\sim} \mathcal{N}(\mu, \sigma^2)$

Conocimiento "exacto"

sepamos el valor de μ y σ

$\rightarrow \mu = 10€ \quad \sigma = 2€$

$\bar{\underline{X}} \sim \mathcal{N}(10, \frac{4}{12})$